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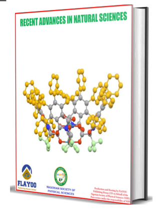

Simulation of optical fiber power amplification via higher-order soliton generation using the extended nonlinear Schrödinger equation

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ABSTRACT

This study seeks to simulate the amplification of the optical fiber power by the generation of higher order solitons. This was achieved by solving an extended form of the nonlinear Schrödinger equation (NLSE) using the split step Fourier method (SSFM) with Gaussian functionals having multiple peaks as initial conditions to simulate the generation of higher order solitons in the optical fiber. Some key fiber parameters such as the coefficients of loss α, group velocity dispersion β and nonlinearity γ were varied and the respective effects on the optical fiber and soliton power were then observed using spatial plots, 3-D contour plots and image color maps. Results obtained showed that in all soliton orders, higher order solitons were created when β was increased from 0.05 to 0.1 fs/nm.km. This shows a broadening of the soliton to create higher order solitons when dispersion is managed within that range which results in a boost in the peak soliton power amplified from 4.5309 W to 14.9508 W and then to 15.0828W as the soliton order was increased from 1^{st} – 2^{nd} – 3^{rd} order respectively using Gaussian functionals. The extra power gained is as a result of the fact that a newly created soliton takes its energy from the radiation present in the dispersed soliton even though the optical power attenuates. It was also observed that, increasing the coefficients α, β and γ from 0.1 – 1.0 results in a continuous attenuation of the optical fiber power leading to the propagation of radiation (noisy signal) in the optical fiber which scatters and exponentially decays after a short distance along the length of the optical fiber.

1. INTRODUCTION

An Optical fiber is a thin and flexible strand of glass or plastic material that transmits information in the form of light pulses. It comprises of three components: the central part called the core, the outer coating material with a lower index of refraction called the cladding and the external protective layer. Light which propagates through the optical fiber is transmitted through the core by the process of total internal reflection which depicts the fiber as a wave guide. Optical fibers are of two types: multi-mode and single mode fibers. Multi-mode fibers support multiple propagation paths or transverse modes while single mode fibers support just

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a single propagation path [1]. Optical fibers are generally better for signal transmission than electric wire cables because light in fibers suffers extremely low power loss and attenuation (about 0.2 dB/km) and fibers does not create external electromagnetic fields thus providing an extremely wide bandwidth [2]. With the ever-growing data traffic which requires more data-carrying capacity and band width heralded by the introduction of more complex and higher capacity networks such as 3G, 4G, and 5G technology, optical fibers are the best medium to curtail the multiplicity of dispersion effects in signal propagation [3]. Optical fiber also has the capacity to transmit data at extremely high speeds (greater than 40 Terabits per second) and thus improves the efficiency of 5G network, sparking a revolution in the digital world [4]. The collaboration between 5G networks and optical fiber has emerged as a crucial catalyst for advancement in communication technology due to the fact that optical fiber facilitates the availability of higher bandwidth for transmission of large amounts of data, reduces latency (network response time), is more energy efficient as it minimizes losses via heating and electromagnetic interference and offers unprecedented scalability while providing higher level of data security i.e. immunity from electronic eavesdropping and data interception [4, 5]. Optical fiber technology is also cost effective on the long run because after initial cost of installation, there is lower maintenance cost and reduced need for frequent repairs and replacement of worn out components as it is not affected by atmospheric attenuation enabling long distance communication [5].

Another advantage of optical fibers over electrical cables is in the mode of propagation; for instance, the response of glass to a light pulse is nonlinear, whereas that of copper to an electric current is linear. This nonlinearity observed in optical fibers has been harnessed via the adoption of the soliton concept [6]. Solitons in optical fibers are pulses of light that stabilize a balance between linear and nonlinear distortions [7]. These light pulses in optical fibers transmit the bulk of all telecommunication signals in recent times. They are localized nonlinear waves which maintain a constant shape and speed throughout its propagation despite the effects of external perturbations. Order wise, solitons can be categorized into two: fundamental and higher order solitons. Fundamental solitons are solitons of order 1; they are the simplest and most stable. Higher order solitons are solitons of order greater than 1. They are composed of overtones of the fundamental soliton, are less stable and exhibit complex behavior. The energy of higher order solitons is higher than that of the fundamental soliton by a factor which is the square of an integer i.e. 1, 4, 9, ... They are utilized for nonlinear pulse compression and degenerate back to the fundamental soliton under effects such as: Raman scattering and higher order dispersion [8]. This instability arises as a result of their periodic shape variations making them vulnerable to perturbations that can disrupt the equilibrium maintained by the nonlinear effects and pulse broadening [9]. Thus in this work, simulations will be undertaken using functionals in the solution of the extended Nonlinear Schrödinger Equation (NLSE) in order to boost the soliton content of an optical fiber and thus amplify the soliton power of the optical fiber and minimize signal loss and dispersion.

2. METHODOLOGY

2.1. THE EXTENDED NONLINEAR SCHRÖDINGER EQUATION

he nonlinear Schrödinger equation (NLSE) is usually applied to obtain the complex amplitude $U(z,t)$ of the soliton propagating through an optical fiber. Its basic form is given as Ref. [7]:

$$i\frac{\partial}{\partial Z}u - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}u + \gamma|A|^2u = 0, \tag{1}$$

where $u(z,t)$ is the complex amplitude of the soliton, α is the loss coefficient in dB/km, β_2 is the coefficient of group velocity dispersion in $fs/(nm.km)$ and γ is the nonlinearity coefficient in m^2/W. However, the basic form of the NLSE neglects linear loss, higher order dispersion parameters and ignores other effects such as Raman scattering (T_R). Hence there is the need to apply the extended NLSE in order to capture more effects. The general form of the extended NLSE is given by Ref. [10]:

$$i\frac{\partial}{\partial Z}u - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}u + \frac{\beta_4}{24}\frac{\partial^4}{\partial t^4}u + \cdots + \gamma|u|^2u + i\frac{\gamma}{\omega_o}\frac{\partial}{\partial t}\left(|u|^2u\right) - T_R u\frac{\partial}{\partial t}|u|^2 + i\frac{\alpha}{2}u = 0, \tag{2}$$

where, $\frac{\beta_4}{24}\frac{\partial^4}{\partial t^4}u$ is the higher order dispersion term. Raman scattering stems from the inelastic scattering of light as it interacts with the fiber's molecules transferring energy between them and causing a shift in the wavelength of the scattered light. Raman scattering effect also boosts the power on the long run [10] and so in this work the effect of Raman scattering is neglected in order to reduce the complexity of the system and thus the extended NLSE can be re-written as:

$$-i\frac{\partial u}{\partial Z} = -\beta_2\frac{\partial^2 u}{\partial t^2} + \beta_4\frac{\partial^4 u}{\partial t^4} + \gamma|u|^2u + i\frac{\alpha}{2}u. \tag{3}$$

Dividing through by the complex variable $-i$ and setting the higher order dispersion coefficient $\beta_4 = 1$ because its effect (pulse broadening) is negligible compared to that of the group velocity dispersion β_2 [11], the extended NLSE is reduced to:

$$\frac{\partial u}{\partial Z} = -i\beta_2\frac{\partial^2 u}{\partial t^2} + i\frac{\partial^4 u}{\partial t^4} + i\gamma|u|^2u - \frac{\alpha}{2}u. \tag{4}$$

2.2. SPLIT-STEP FOURIER METHOD

In this work nonlinear partial differential equation was solved by applying the Split-Step Fourier Method (SSFM). First of all, the linear (L) and nonlinear (N) parts of the extended NLSE in Equation (4) are identified as follows:

$$L = -i\beta_2\frac{\partial^2 u}{\partial t^2} + i\frac{\partial^4 u}{\partial t^4} - \frac{\alpha}{2}u \text{ and } N = i\gamma|u|^2u. \tag{5}$$

Then proceed to purely solve the equation for the nonlinear part N, neglecting the linear term, L:

$$\frac{\partial u}{\partial z} = i\gamma|u|^2u. \tag{6}$$

The resultant ordinary differential equation (ODE) is then solved at increments Δt to give the solution:

$$u(z,\ t+\Delta t) = \exp\left(i\gamma|u|^2\Delta t\right)u(z,t). \tag{7}$$

Neglecting N in the equation (4) we have:

$$\frac{\partial u}{\partial z} = (-i\beta_2\frac{\partial^2}{\partial t^2} + i\frac{\partial^4}{\partial t^4} - \frac{\alpha}{2})u. \tag{8}$$

Table 1. List of functionals used as initial conditions for the generation of higher order solitons using the extended NLSE.

Soliton Order	Description of Soliton	Functionals	Remark
1	First order	$u_o = (\text{sech}x\ )^2$ or $u_o = e^{-x^2}$	Fundamental Soliton
2	Second order	$u_o = \ e^{-(x+3)^2} + e^{-(x-3)^2}$	Higher order solitons
3	Third order	$u_o = \ e^{-(x+5)^2} + e^{-x^2} + e^{-(x-5)^2}$	
4	Fourth order	$u_o = \ e^{-(x+6)^2} + e^{-(x+2)^2} + e^{-(x-2)^2} + e^{-(x+6)^2}$	
5	Fifth order	$u_o = \ e^{-(x+8)^2} + e^{-(x+4)^2} + \ e^{-x^2} + e^{-(x-4)^2} + e^{-(x+8)^2}$	

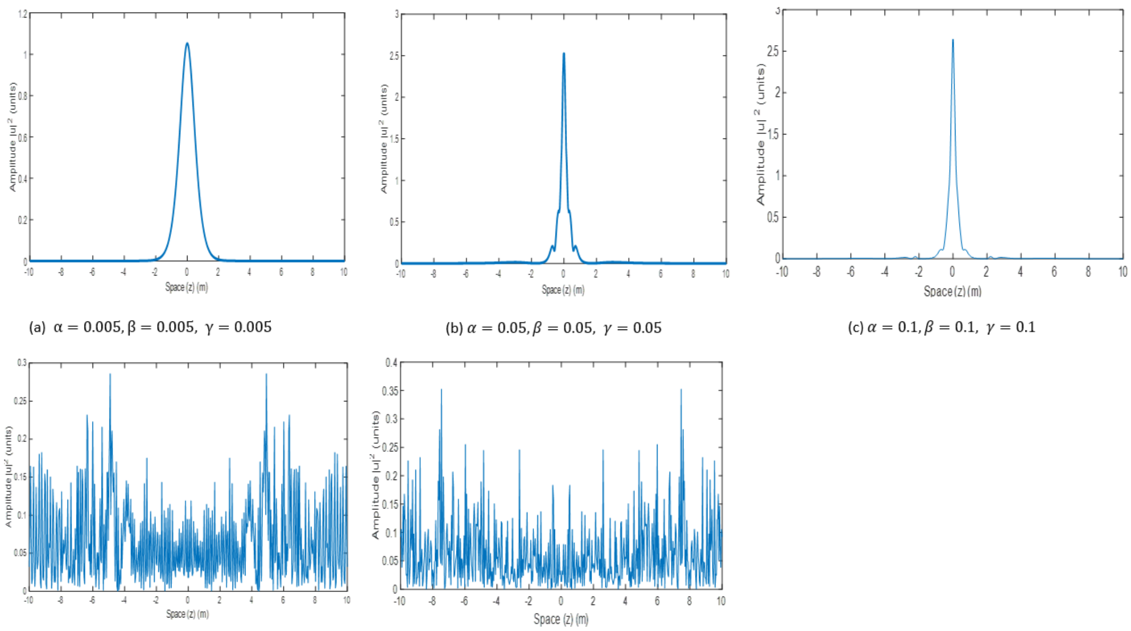
(a) $\alpha = 0.005, \beta = 0.005,\ \gamma = 0.005$
(b) $\alpha = 0.05, \beta = 0.05,\ \gamma = 0.05$
(c) $\alpha = 0.1, \beta = 0.1,\ \gamma = 0.1$
(d) $\alpha = 0.5, \beta = 0.5,\ \gamma = 0.5$
(e) $\alpha = 1.0, \beta = 1.0,\ \gamma = 1.0$

Figure 1. Variation of the amplitude ($|U|^2$)of the first order soliton with space coordinate z, for a range of values of $\alpha,\ \beta\ \&\ \gamma$.

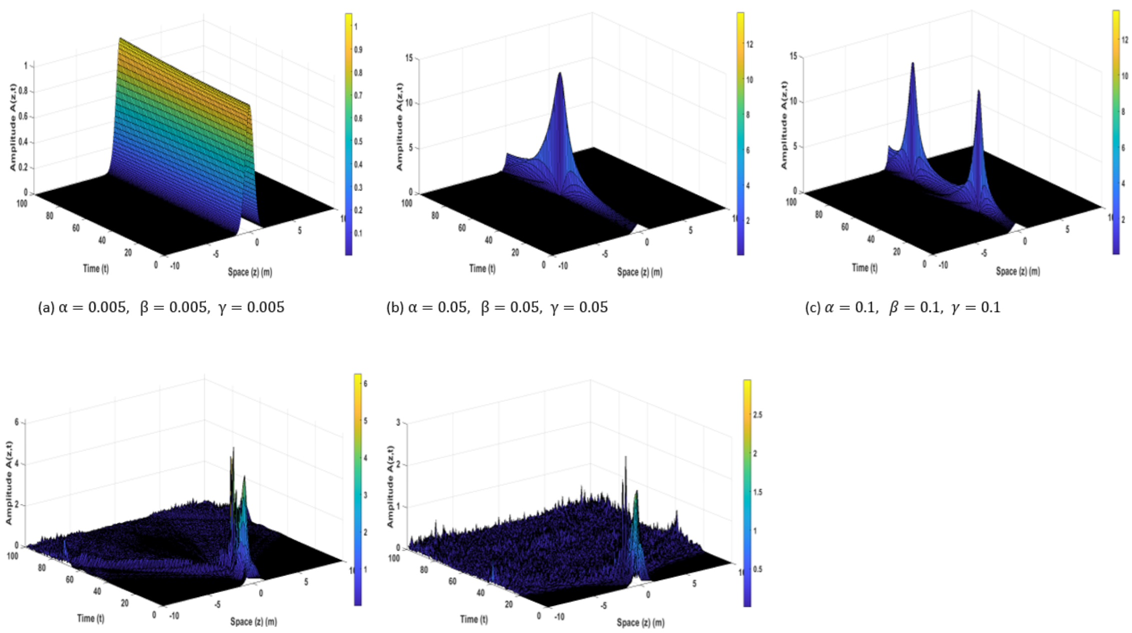
(a) $\alpha = 0.005,\ \beta = 0.005,\ \gamma = 0.005$
(b) $\alpha = 0.05,\ \beta = 0.05,\ \gamma = 0.05$
(c) $\alpha = 0.1,\ \beta = 0.1,\ \gamma = 0.1$
(d) $\alpha = 0.5,\ \beta = 0.5,\ \gamma = 0.5$
(e) $\alpha = 1.0,\ \beta = 1.0,\ \gamma = 1.0$

Figure 2. Contour plot of the amplitude ($|U|^2$)of the first order soliton with time(t) and space (z) for a range of values of $\alpha,\ \beta\ \&\ \ \gamma$.

Solving the resultant ODE in the interval $[t,\ t + \Delta t]$ and use $u\,(x,\ t + \Delta t)$ in equation (5) as initial condition. An analytical

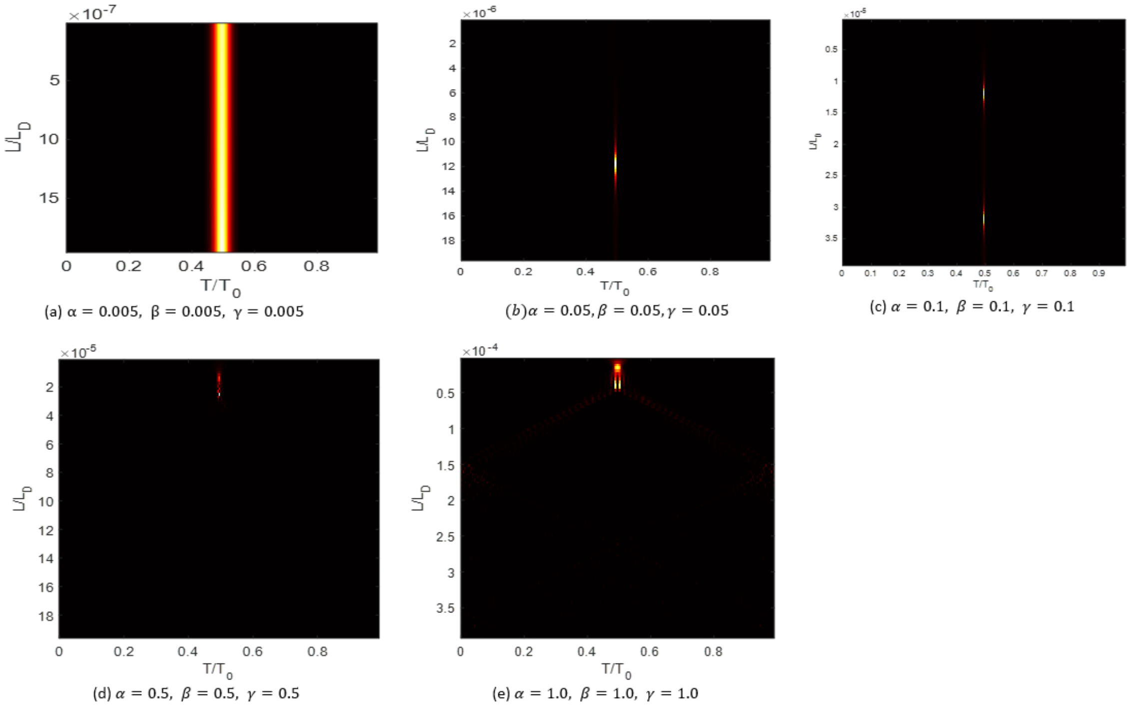

Figure 3. Image color maps of the first order soliton for a range of values of $\alpha,\ \beta\ \&\ \gamma$.

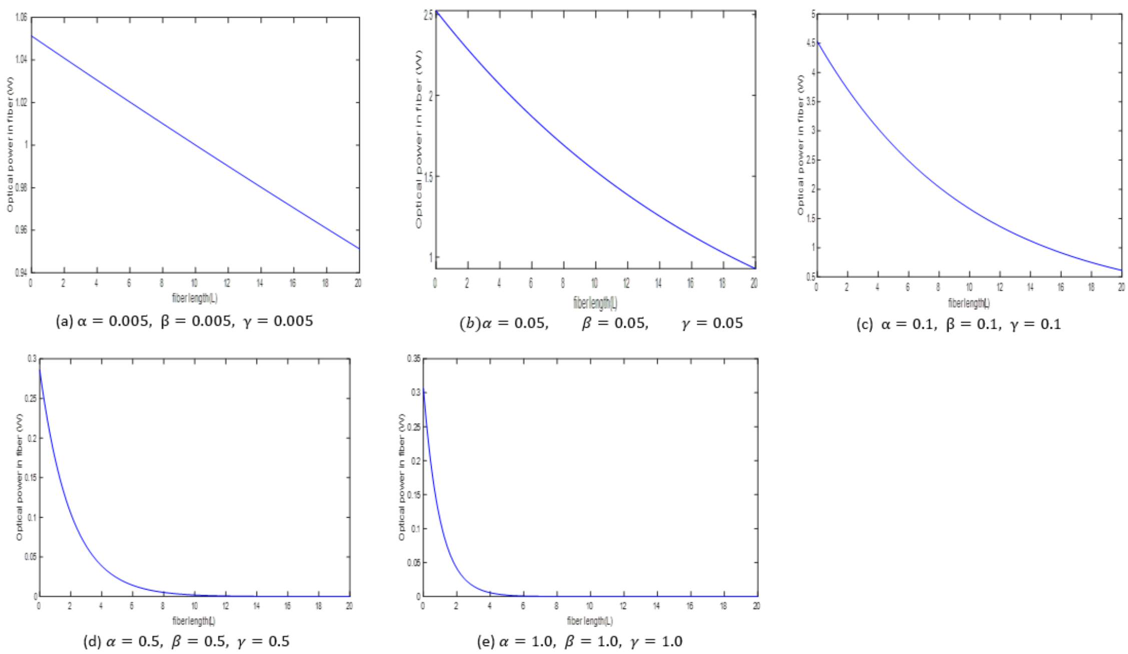

Figure 4. Variation of the optical fiber power of the first order soliton with space coordinate z, for a range of values of $\alpha,\ \beta\ \&\ \gamma$.

solution of equation (8) was then obtained by taking the Fourier transform:

$$\mathcal{F}\left(\frac{\partial U(z,t)}{\partial z}\right) = \hat{u}\,(k,t+\Delta t) = \exp(\mathcal{F}\left(-i\beta_2\frac{\partial^2}{\partial t^2} + i\frac{\partial^4}{\partial t^4} - \frac{\alpha}{2}\right)\Delta t)\hat{u}(k,t). \tag{9}$$

Applying $\mathcal{F}(\frac{\partial}{\partial t}) = i\omega$ gives:

$$\hat{u}\,(k,t+\Delta t) = \exp\left(i\beta\omega^2\Delta t + i\omega^4\Delta t - \frac{\alpha}{2}\right)\hat{u}(k,t). \tag{10}$$

The solution of the z domain can be obtained by taking the inverse Fourier transform in the z domain is given as:

$$\mathcal{F}^{-1}\,(\hat{u}\,(k,t+\Delta t)) = u\,(z,t+\Delta t) = \mathcal{F}^{-1}\left[\exp\left(i\beta\omega^2\Delta t + i\omega^4\Delta t - \frac{\alpha}{2}\right)\hat{u}(k,t+\Delta t)\right].(11)$$

But from equation (6):

$$\hat{u}\,(k,t+\Delta t) = \mathcal{F}\,(u(z,t+\Delta t)) = \ \mathcal{F}\left(\exp\left(i\gamma|u|^2\Delta t\right)u(z,t)\right). \tag{12}$$

Substituting equation (12) into (11), the overall solution can be

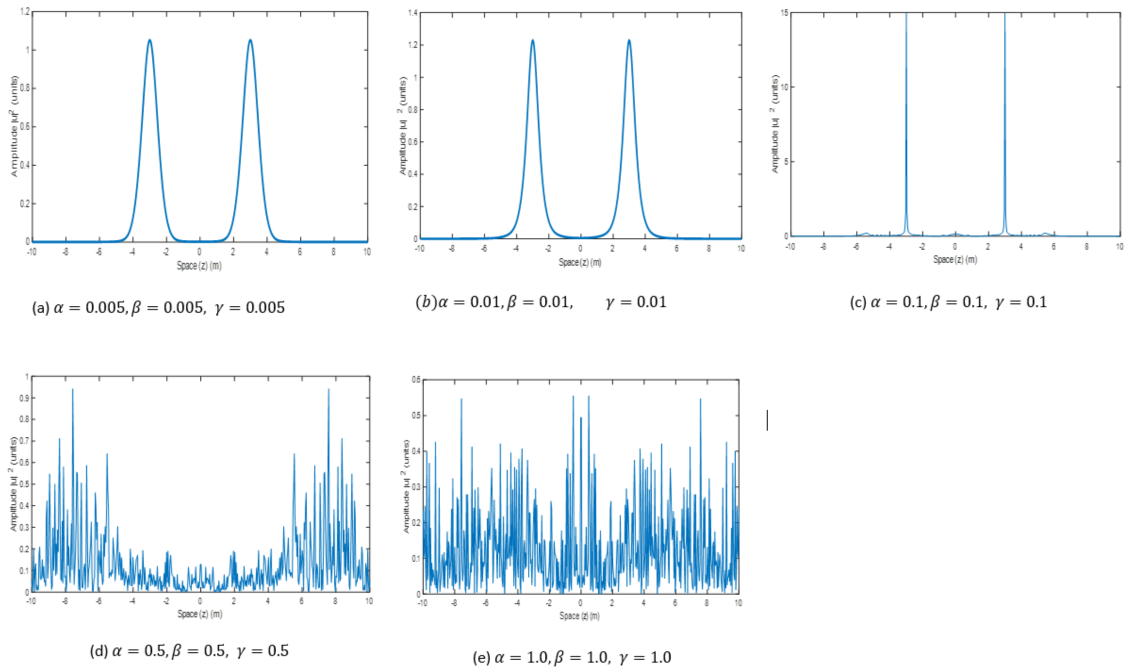

Figure 5. Variation of the amplitude ($|U|^2$)of the second order soliton with space (z) for a range of values of $\alpha,\ \beta\ \&\ \gamma$.

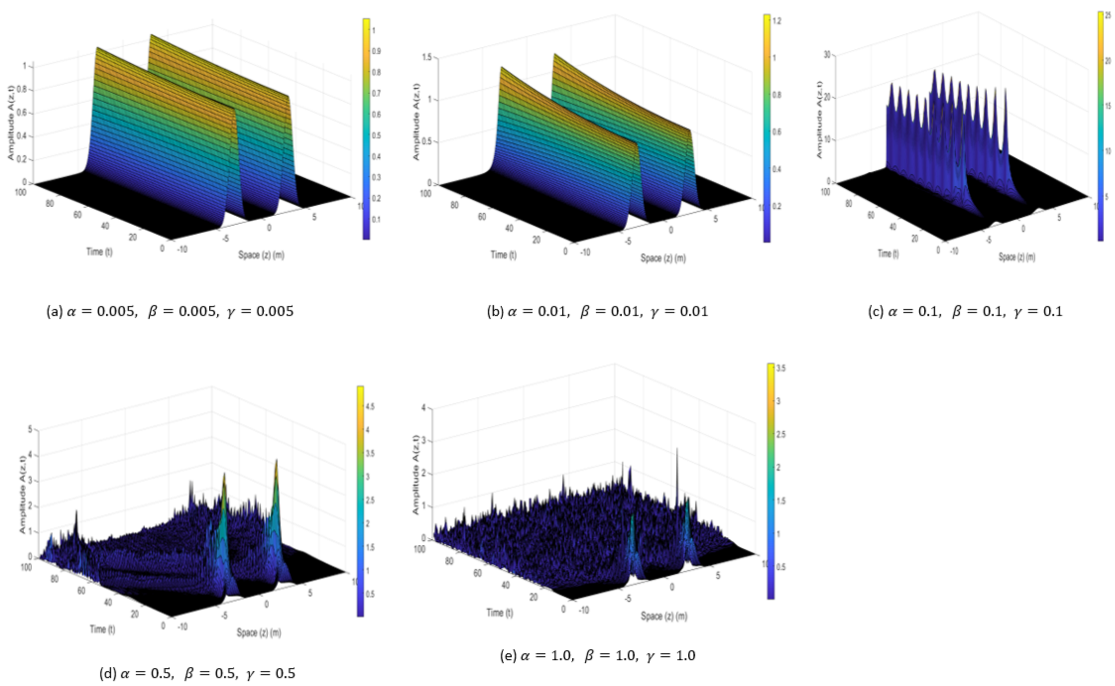

Figure 6. Contour plot of the amplitude ($|U|^2$)of the second order soliton with time(t) and space (z) for a range of values of $\alpha,\ \beta\ \&\ \gamma$.

written in a single equation viz:

$$u(z,\tau+\Delta t)=\mathcal{F}^{-1}\left[\exp\left(i\beta\omega^2\Delta t+i\omega^4\Delta t-\frac{\alpha}{2}\right).\mathcal{F}\left(\exp\left(i\gamma|u|^2\Delta t\right)u(z,t)\right)\right]. \quad (13)$$

The peak power of the soliton is computed from the relation [12]:

$$P_o=\max(|u(z,t)|^2). \quad (14)$$

This is the optical fiber power at the origin. The optical power in a fiber $P(z)$, at a distance z is given by:

$$P(z)=P_o\exp(-\alpha z). \quad (15)$$

The soliton power on the other hand is given by Ref. [10]:

$$P(t)=P_o sech^2\left(\frac{t}{T_o}\right), \quad (16)$$

where T_o is the pulse duration. The attenuation coefficient,

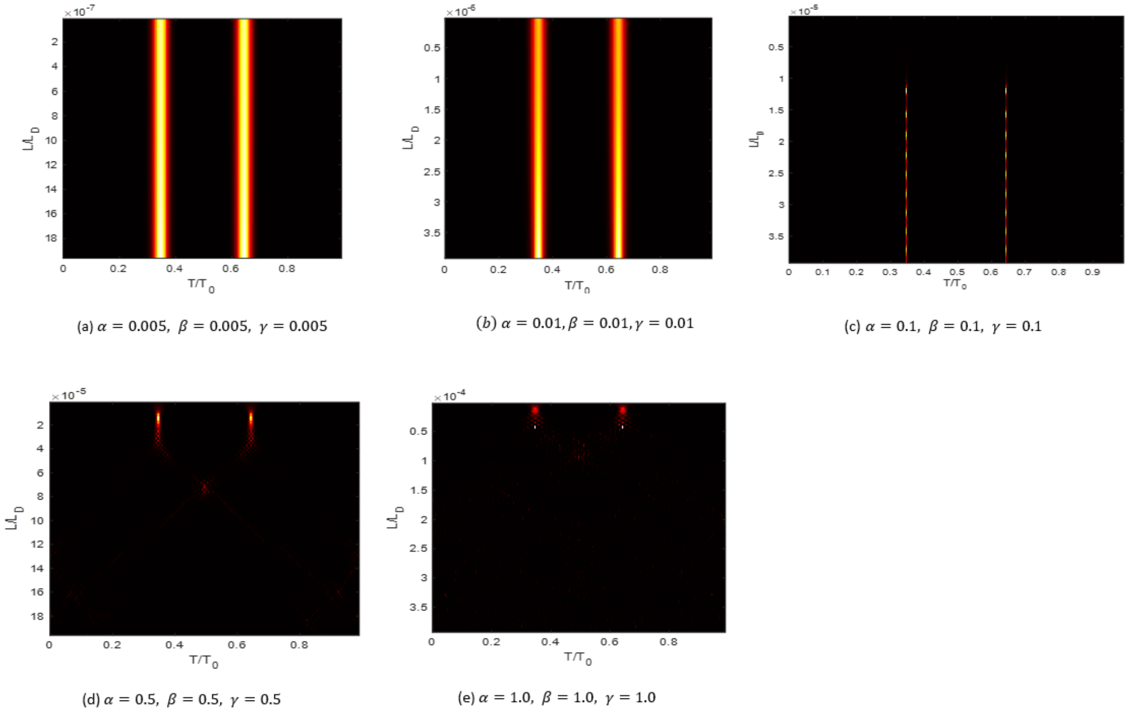

Figure 7. Image color maps of the second order soliton for a range of values of $\alpha,\ \beta\ \&\ \gamma$.

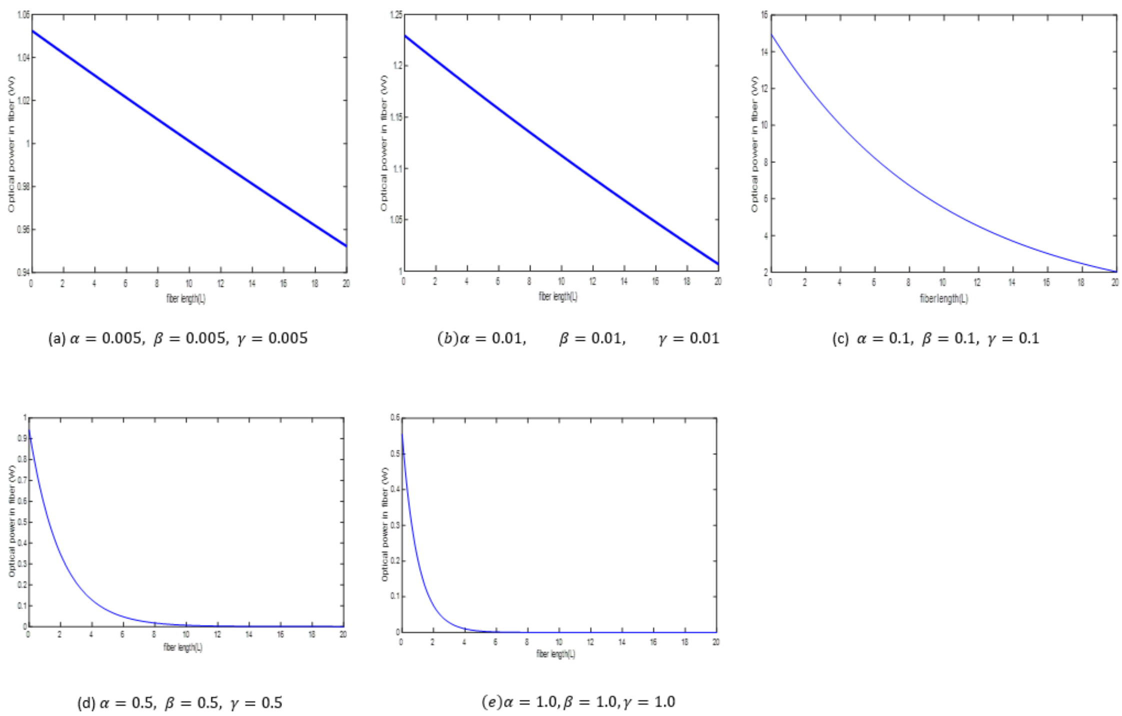

Figure 8. Variation of the optical fiber power of the second order soliton with fiber length (L) for a range of values of $\alpha,\ \beta\ \&\ \gamma$.

α *per km*, is given by the relation:

$$\alpha = \frac{10}{z}\log\left[\frac{P_o}{P(z)}\right]. \tag{17}$$

Without the effects of nonlinearity γ, the shape of the light pulse could be greatly distorted after a unit of dispersion length L_D [12]. L_D was computed using the expression:

$$L_D = \frac{T_o^2}{|\beta|} \tag{18}$$

The nonlinearity length L_N enables soliton self-phase modula-

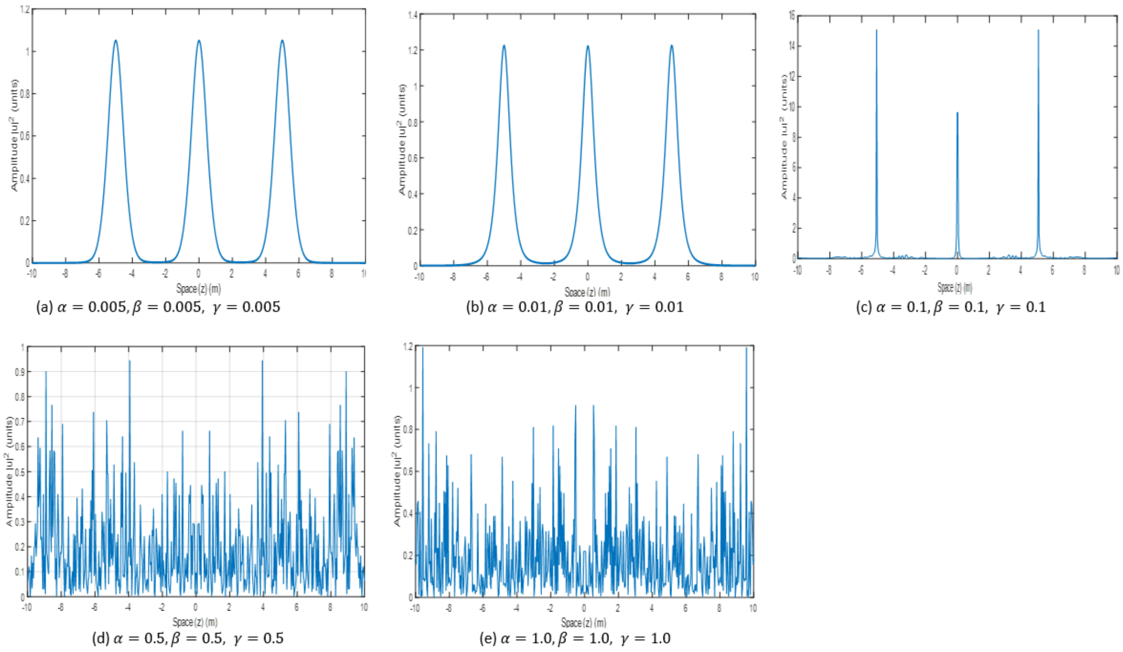

Figure 9. Variation of the amplitude ($|U|^2$) of the third order soliton with space (z) for a range of values of $\alpha,\ \beta\ \&\ \gamma$.

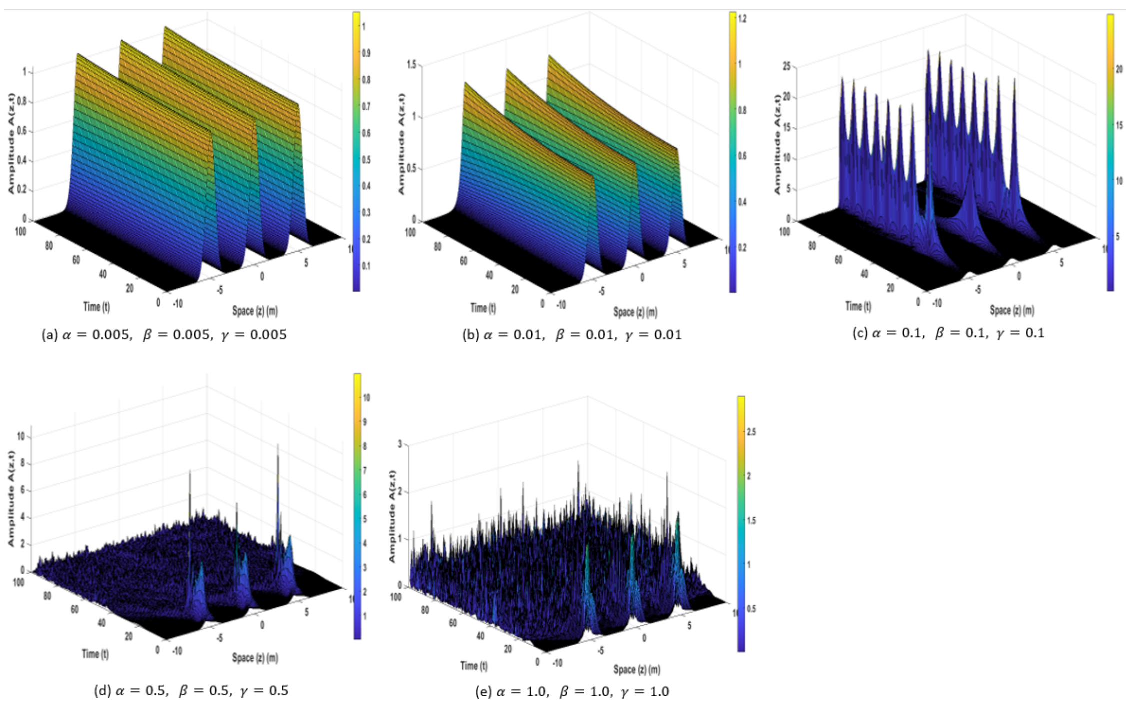

Figure 10. Contour plot of the amplitude ($|U|^2$) of the third order soliton with time(t) and space (z) for a range of values of $\alpha,\ \beta\ \&\ \gamma$.

tion and is expressed as:

$$L_N = \frac{1}{\gamma P_o}. \tag{19}$$

Also as a result of this phenomenon of self-phase modulation, there must exist a balance between β_2 *and* γ, such that: $L_D = L_N$ and thus, the complex amplitude, u and pulse duration must satisfy the condition [12]:

$$P_o {T_o}^2 = \frac{|\beta_2|}{\gamma}. \tag{20}$$

Table 1 shows the exponential and hyperbolic functionals developed as the initial boundary conditions to facilitate the generation of higher order solitons for the propagation of solitons in an optical fiber through the extended NLSE.

In our simulation, the fiber length $L = 20\ km$, pulse duration T_o = 100 fs and the number of spatial grid points N = 512. The V-parameter was set > 2.405 (single mode fiber) [13], while the

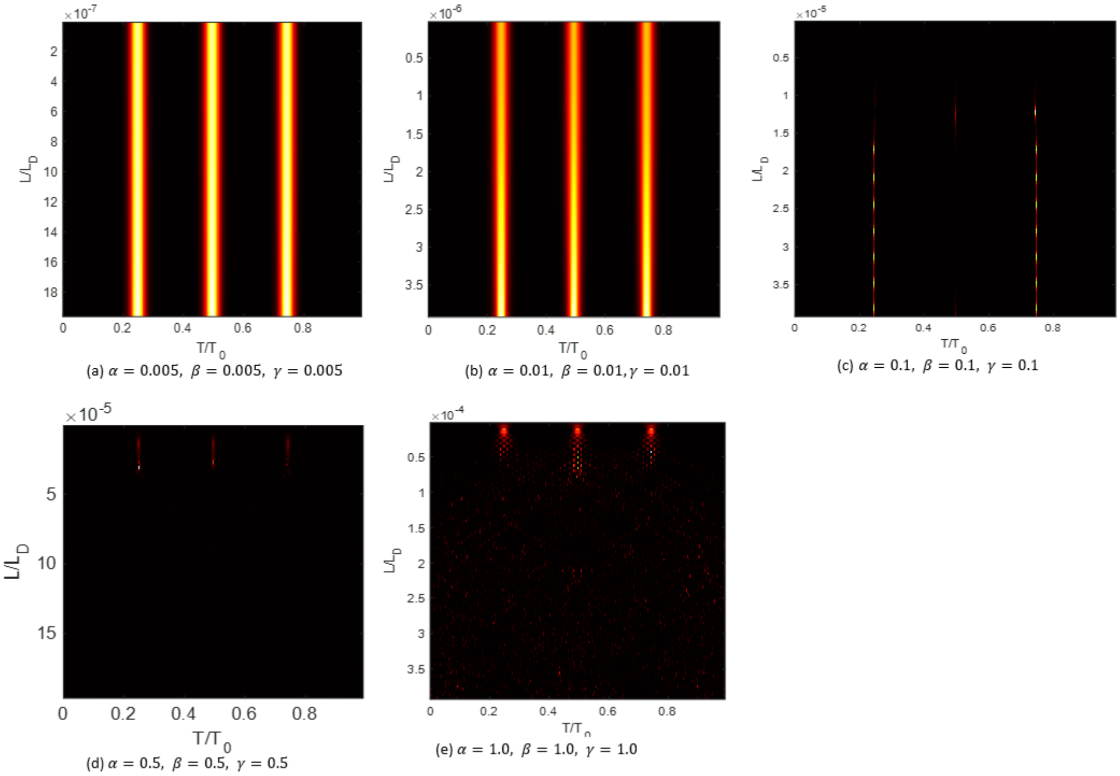

Figure 11. Image color maps of the third order soliton for a range of values of $\alpha,\ \beta\ \&\ \gamma$.

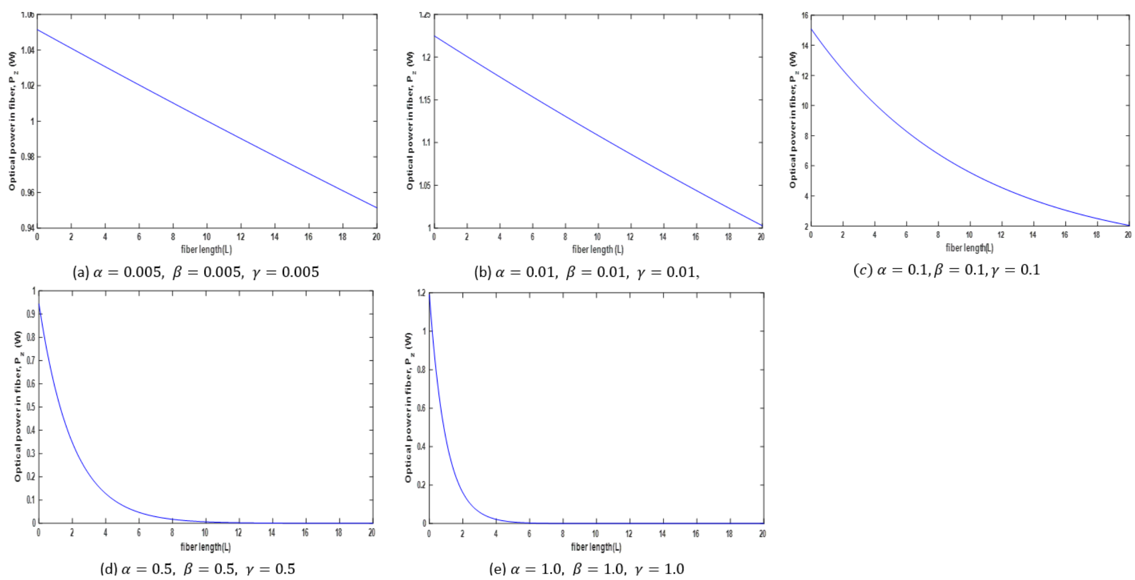

Figure 12. Variation of the optical fiber power of the third order soliton with fiber length (L) for a range of values of $\alpha,\ \beta\ \&\ \gamma$.

frequency vector ω (spatial frequencies) was set as Ref. [14]:

$$\omega = \frac{2\pi}{L}\left[0 : \frac{N}{2} - 1, -\frac{N}{2} : -1\right]. \tag{21}$$

The parameters $\alpha\ (dB/km), \beta(fs/(nm.km)\ and\ \gamma(m^2/W)$ were randomly varied from 0.005 to 1.0 and the resultant soliton peak power in the optical fiber was computed.

3. RESULTS AND DISCUSSION

The results presented in Figures 1-12 show the soliton solutions of the extended (NLSE) using the functionals in Table 1 for the first order (i.e. fundamental) and higher order solitons. The results illustrate the variation of the square of the complex amplitude $|u|^2$ (soliton power) with space coordinate, z and time, t. It can be observed that the amplitude of the soliton maintains a single peak when the loss, dispersion and nonlinearity coefficients are very low but exhibits multiple peaks when the value of these coefficients are significantly increased. This is visualized in the

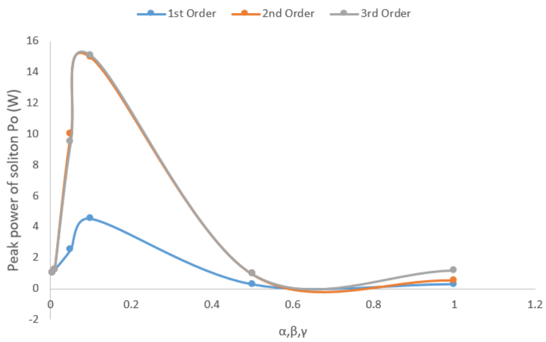

Figure 13. Variation of soliton peak power, P_o with fiber parameters for the 1^{st}, 2^{nd} and 3^{rd} Order solitons.

image color maps plotted, where the single illuminated orange line shows the soliton pulse propagation inside the optical fiber. It is also observed that under a high increase in loss coefficient, a faded image of the soliton being propagated is displayed through the optical fiber. The result in Figure 4 also shows that attenuation of the optical power with fiber length with increase in loss coefficent, α. This trend is also observed when higher order solitons are generated and their key parameters varied.

The fundamental soliton is one which corresponds to order one (1) and is the most stable solution of the NLSE. It transmits at peak power and also has a short pulse duration. The fundamental soliton in this work was generated from a *sech* pulse as shown in Table 1. From the results in Figures 1-4, the behavior of the fundamental soliton was simulated by varying the three main parameters: α, β and γ. From the results obtained, it was observed that when the three parameters are set very low (i.e. $\alpha = \beta = \gamma = 0.005$), the soliton peak power value of about 1.053 W is obtained. As the parameters are varied up to 0.1, the optical fiber power peaks at 4.5309 W. This is because an increase in dispersion up to 0.1 causes a split in the soliton order from first to second orders which in turn increases the soliton content and so the constructive interference that occurs boosts the fundamental soliton optical power. In figure 13 it was observed that the soliton peak power in the optical fiber rises from 1.053 – 4.5309 W, as the fiber parameters are increased from 0.005 to 0.1, then it begins to drop for values > 0.1 until it approaches zero at $\alpha = \beta = \gamma = 0.6$. It was also observed that the soliton peak power attenuates steadily (exponentially decays) with increase in fiber length as all three fiber parameters are steadily increased. This results in the broadening and fading of the signal as it propagates along the fiber length as shown in Figure 4(a-d). This is the worst-case scenario and it leads to the total damping and scattering of the fundamental soliton in the optical fiber.

Higher order solitons are localized pulses with multiple peaks that maintain their shapes over shorter distances in nonlinear optical fibers. They exhibit periodic and quasi periodic behaviors and usually breakdown into fundamental solitons when distorted. They are overtones or harmonics of the fundamental soliton and are formed by nonlinear phase shifts in high intensity pulses, dispersion management and application of an initial chirp to the fundamental soliton. In this simulation, higher order solitons were generated using Gaussian functions with multiple smooth peaks as initial conditions to the NLSE. Second order solitons were created by applying a function with two Gaussian peaks as illustrated in Table 1. The results presented in Figures 5-8 shows the behavior of the second order soliton under the variation of the fiber parameters α, β *and* γ. Here too, it was observed that when the three parameters are set at a very low value (i.e. $\alpha = \beta = \gamma = 0.005$), the soliton peak power was observed to be $P_o = 1.053\ W$. But as the three fiber parameters are increased steadily up to 0.1, the optical fiber soliton power gets amplified and peaks at 14.9508 W. This is because an increase in the value of β up to 0.1 causing a split in the soliton order from second to third order which in turn increases the soliton content and thus the constructive interference that occurs boosts the fundamental soliton optical power. In Figure 13 it can be seen that the peak power of the optical fiber rises from 1.053 – 14.9508 W as the fiber parameters are increased from 0.005 to 0.1, then it begins to drop for values > 0.1 until it hits zero at $\alpha = \beta = \gamma = 0.6$. Beyond the critical value of 0.6, the soliton peak power attenuates and radiation (noisy signal) is formed, leading to a total decay of the signal along the fiber length as illustrated in Figure 8 (a-d). In the results in Figures 5(c & d) to 8(c & d) there is a complete destruction of the soliton order, fading, pulse broadening and attenuation of the peak power as all three parameters alpha α, β *and* γ are increased progressively. It was also observed that, beyond $\alpha = \beta = \gamma = 0.1$, the soliton order is destroyed resulting in propagation of radiation in the optical fiber which broadens and attenuates after a very short distance along the length of the optical fiber [15]. The image color maps in Figure 7(a-d) confirms this phenomenon.

Furthermore, the third order solitons were generated by using a Gaussian function with three Gaussians peak as initial conditions in solving the extended nonlinear Schrödinger equation. The results in Figures 9-12 illustrate the effect of increasing α, β and γ simultaneously from 0.005 - 1.0. it was observed that when the three parameters are very low (i.e. $\alpha = \beta = \gamma = 0.005$), the soliton peak power value computed was found to be 1.053 W. As the parameters were varied up to 0.1, the optical fiber soliton power got amplified and peaked at 14.9508 W. This is because an increase in dispersion up to 0.1 causes a split in the soliton order from second to third order which in turn increased the soliton content in the optical fiber and so the constructive interference that occurred boosted the fundamental soliton optical power. In Figure 13 it can be seen that the peak power of the optical fiber rose from 1.053 – 15.0828 W, slightly above the value of 14.9508 W recorded for the second order soliton as the fiber parameters are increased from 0.005 to 0.1. Beyond this threshold value of $\alpha = \beta = \gamma = 0.1$, the soliton peak power began to drop until a total attenuation and broadening along the fiber length occurs at $\alpha = \beta = \gamma = 0.6$ just like in the first and second order soliton. At parameter values greater than 0.6, the third order soliton order degenerates into radiation in the optical fiber which attenuate and scatters after a short distance along the fiber length. The image color maps in Figure 11(a-d) illustrates this phenomenon.

In summary, the results of this simulation shows that the soliton peak power was amplified from 4.5309 W to 14.9508W and then to 15.0828W as the soliton order was increased from 1^{st} – 2^{nd} – 3^{rd} order respectively using Gaussian functionals. The extra power gained is as a result of the fact that, a newly created soliton takes its energy from the radiation present in the dispersed soliton which serves as a boost for signal transmission via wave propagation through the optical fiber [16]. These results can be applied to amplify transmission lines or in fiber lasers.

4. CONCLUSION

The research has demonstrated the application of the extended NLSE as a modeling framework in capturing the complexities of soliton dynamics in optical fibers. The findings reveal that, by generating higher order solitons using Gaussian functionals, the optical fiber power could be amplified from 4.5309 W for the fundamental soliton to 14.9508W in the second order and 15.0828W in third order soliton. The extra power gained is as a result of the fact that, a newly created soliton takes its energy from the radiation present in the dispersed soliton which serves as a boost for signal transmission via wave propagation through the optical fiber and with the right control of the dispersion and nonlinearity, the soliton content could be optimized and stabilized. The research adds to the body of knowledge for the design of more advanced optical networks to meet up the ever growing bandwidth and high speed data (5G) communication demands.

DATA AVAILABILITY

We do not have any research data outside the submitted manuscript file.

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