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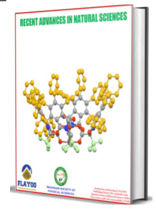

Numerical method for simulation of quadratic Riccati differential equations

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ABSTRACT

Differential equations widely applied in various fields of engineering and mathematical studies, particularly in control theory, optimal control, and stochastic realization. Numerous methods have been proposed for their solution. Although QRDEs pose significant analytical challenges, several approaches such as the Variational Iteration Method, Differential Transform Method, Runge-Kutta methods, and Non-Standard Finite Difference Methods have been developed to address them. In this paper, we present a computational method for solving QRDEs by employing the collocation technique in combination with a power series approximation. Fundamental properties of the method, including order, consistency, zero-stability, and convergence, are analyzed. Numerical simulations demonstrate that the proposed method provides accurate and stable results, showing strong agreement with existing methods when applied to different QRDE models. Furthermore, the findings highlight the efficiency and applicability of the approach in solving nonlinear equations of considerable complexity.

1. INTRODUCTION

The quadratic Riccati differential equations (QRDEs), a class of ordinary differential equations (ODEs), arise in various fields, including mathematics, physics, and engineering Batiha [1]. In this study, we consider the QRDE of the form:

$$y'(t) = a(t)y(t) + b(t)y^2(t) + c(t), \tag{1}$$

with initial conditions:

$$y(t_0) = y_0, \tag{2}$$

where $y(t)$ is the unknown function, and $a(t), b(t)$ and $c(t)$ are continuous functions with $c(t) \neq 0$ and t_0, y_o are arbitrary constants of t.

The term $b(t)y^2(t)$ named for its quadratic term, this equation poses analytical challenges, often lacking closed-form solution. Nonetheless, its relevance spans across control theory, optimal control, stability analysis and related fields. Quadratic Riccati Differential Equations (QRDEs) find extensive applications across diverse scientific and engineering domains due to their ability to model nonlinear and dynamic systems [2]. In control theory, they play a pivotal role in solving optimal control problems and designing robust control systems [3]. They are also central to stochastic realization theory, where they are used for filtering and estimation processes [4, 5]. Furthermore, QRDEs have been applied in modeling physical phenomena such as diffusion in electrical circuits and the bending of beams, demonstrating their versatility in describing physical systems Opanuga [6]. In mathematical physics, they provide solutions to nonlinear partial differential equations, including solitary wave equations

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expressed in terms of elementary functions satisfying projective Riccati equations [3]. Beyond the traditional sciences, QRDEs have also gained importance in financial mathematics, particularly in option pricing and risk assessment, underscoring their relevance in both classical and emerging fields [7].

Several analytical and numerical methods have been developed to solve QRDEs. The Variational Iteration Method (VIM) has been introduced as a powerful analytical tool for both linear and nonlinear differential equations, including QRDEs, offering iterative refinement for improved accuracy [8]. Its effectiveness and convergence have been validated through numerous applications [9]. An extension of this approach, the Modified Variational Iteration Method (MVIM), incorporates multiple stages of approximation refinement to enhance accuracy when dealing with the increased computational complexity of QRDEs [10]. Other widely used methods include the Differential Transform Method (DTM), which simplifies differential equations into algebraic forms for easier computation as stated in Kamoh [11]; the Classical Fourth-Order Runge-Kutta Method, well-regarded for its simplicity and reliability in solving ODEs Kumleng [12]; and the Chebyshev Wavelet Method, which employs orthogonal functions for accurate representation and solution of QRDEs [5, 13]. Additional approaches, such as the Adomian Decomposition Method in Al-Din [3, 4], the Hybrid Functions and Tau Method, and the Non-Standard Finite Difference Method, further highlight the wide applicability and robustness of existing techniques in addressing the challenges of solving QRDEs [14].

2. DERIVATION OF THE COMPUTATION METHOD

The approximate solution is assumed in the form of a power series polynomial, expressed as:

$$y(t) = \sum_{j=0}^{r+s-1} a_j t^j, \tag{3}$$

where r and s denote the numbers of collocation and interpolation points, respectively. Suppose we represent the approximate solution to Eq. (1) with a power series of degree 7, this entails allowing $r + s - 1 = 7$ in Eq. (3) becomes:

$$y(t) = \sum_{j=0}^{7} a_j t^j = a_0 + a_1 t + a_2 t^2 + \cdots + a_7 t^7, \tag{4}$$

with the first derivative of Eq. (4) given by:

$$y'(t) = \sum_{j=0}^{7} j a_j t^{j-1} = a_1 + 2a_2 t + 3a_3 t^2 \cdots + 7a_7 t^6. \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (1) yields:

$$f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots + a_7 y^7, \tag{6}$$

$$f'(t) = a_1 + 2a_2 t + 3a_3 t^2 \cdots + 7a_7 t^6. \tag{7}$$

Now, interpolating Eq. (6) at point $t_{n+s}, s = 1$ and collocating Eq. (7) at points $t_{n+r}, r = 0\left(\frac{1}{6}\right)1$, leads to a system of nonlinear equation of the form:

$$XA = Y, \tag{8}$$

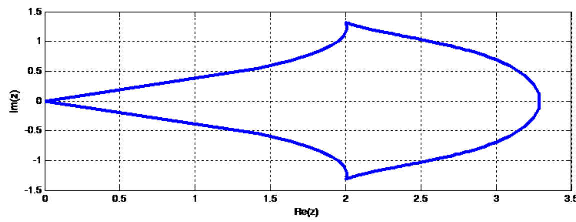

Figure 1. Region of stability.

where $A = [a_0\ a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7]$, *and* $Y = \left[y_{n+1}\ f_n\ f_{n+\frac{1}{6}}\ f_{n+\frac{1}{3}}\ \ f_{n+\frac{2}{3}}\ f_{n+\frac{5}{6}}\ f_{n+1}\right]^T$,

$$X = \begin{bmatrix} 1 & t_{n+1} & t_{n+1}^2 & t_{n+1}^3 & t_{n+1}^4 & t_{n+1}^5 & t_{n+1}^6 & t_{n+1}^7 \\ 0 & 1 & 2t_n & 3t_n^2 & 4t_n^3 & 5t_n^4 & 6t_n^5 & 7t_n^6 \\ 0 & 1 & 2t_{n+\frac{1}{6}} & 3t_{n+\frac{1}{6}}^2 & 4t_{n+\frac{1}{6}}^3 & 5t_{n+\frac{1}{6}}^4 & 6t_{n+\frac{1}{6}}^5 & 7t_{n+\frac{1}{6}}^6 \\ 0 & 1 & 2t_{n+\frac{1}{3}} & 3t_{n+\frac{1}{3}}^2 & 4t_{n+\frac{1}{3}}^3 & 4t_{n+\frac{1}{3}}^4 & 6t_{n+\frac{1}{3}}^5 & 7t_{n+\frac{1}{3}}^6 \\ 0 & 1 & 2t_{n+\frac{1}{2}} & 3t_{n+\frac{1}{2}}^2 & 4t_{n+\frac{1}{2}}^3 & 4t_{n+\frac{1}{2}}^4 & 6t_{n+\frac{1}{2}}^5 & 7t_{n+\frac{1}{2}}^6 \\ 0 & 1 & 2t_{n+\frac{2}{3}} & 3t_{n+\frac{2}{3}}^2 & 4t_{n+\frac{2}{3}}^3 & 4t_{n+\frac{2}{3}}^4 & 6t_{n+\frac{2}{3}}^5 & 7t_{n+\frac{2}{3}}^6 \\ 0 & 1 & 2t_{n+\frac{5}{6}} & 3t_{n+\frac{5}{6}}^2 & 4t_{n+\frac{5}{6}}^3 & 5t_{n+\frac{5}{6}}^4 & 6t_{n+\frac{5}{6}}^5 & 7t_{n+\frac{5}{6}}^6 \\ 0 & 1 & 2t_{n+1} & 3t_{n+1}^2 & 4t_{n+1}^3 & 5t_{n+1}^4 & 6t_{n+1}^5 & 7t_{n+1}^6 \end{bmatrix}.$$

By applying the Gauss elimination method to solve the system of nonlinear equations for the unknown variables and substituting the resulting coefficients into the power series basis function, we obtain a one-step block method with five off-mesh points, expressed in the following form:

$$y(t) = \alpha_1(t) y_{n+1} + h \sum_{j=0}^{1} \beta_j(t) f_{n+j}, \quad j = 0\left(\frac{1}{6}\right)1, \tag{9}$$

where the coefficients of y_n and f_{n+j} are given as:

$$\left.\begin{aligned} \alpha_1 &= 1, \\ \beta_0 &= -\frac{41}{840} + t - \frac{147}{20}t^2 + \frac{406}{15}t^3 - \frac{441}{8}t^4 + 63t^5 - \frac{189}{5}t^6 + \frac{324}{35}t^5, \\ \beta_{\frac{1}{6}} &= -\frac{9}{35} + 18t^2 - \frac{522}{5}t^3 + 261t^4 - \frac{1674}{5}t^5 + 216t^6 - \frac{1944}{35}t^5, \\ \beta_{\frac{1}{3}} &= -\frac{9}{280} - \frac{45}{2}t^2 + \frac{351}{2}t^3 - \frac{4149}{8}t^4 + \frac{3699}{5}t^5 - 513t^6 - \frac{972}{7}t^7, \\ \beta_{\frac{1}{2}} &= -\frac{34}{105} + 20t^2 - \frac{504}{3}t^3 + 558t^4 - \frac{4356}{5}t^5 + 648t^6 - \frac{1296}{7}t^7 \\ \beta_{\frac{2}{3}} &= -\frac{9}{280} - \frac{45}{4}t^2 + 99t^3 - \frac{2763}{8}t^4 + \frac{2889}{5}t^5 - 459t^6 - \frac{972}{7}t^7 \\ \beta_{\frac{5}{6}} &= -\frac{9}{35} + \frac{18}{5}t^2 - \frac{162}{5}t^3 + 117t^4 - \frac{1026}{5}t^5 + \frac{864}{5}t^6 - \frac{1944}{35}t^7 \\ \beta_1 &= -\frac{41}{840} - \frac{1}{2}t^2 + \frac{137}{30}t^3 - \frac{135}{8}t^4 + \frac{135}{5}t^5 - 27t^6 + \frac{324}{35}t^7. \end{aligned}\right\} \tag{10}$$

and x is given by Eq. (10). Evaluating Eq. (9) at $t = \frac{1}{6}\left(\frac{1}{6}\right)1$, gives the new discrete method as:

$$\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+\frac{1}{6}}\\y_{n+\frac{1}{3}}\\y_{n+\frac{1}{2}}\\y_{n+\frac{2}{3}}\\y_{n+\frac{5}{6}}\\y_{n+1}\end{bmatrix}=\begin{bmatrix}0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n-\frac{1}{6}}\\y_{n-\frac{1}{3}}\\y_{n-\frac{1}{2}}\\y_{n-\frac{2}{3}}\\y_{n-\frac{5}{6}}\\y_{n}\end{bmatrix}$$

$$+h\begin{bmatrix}0&0&0&0&0&\frac{19087}{362880}\\0&0&0&0&0&\frac{1139}{22680}\\0&0&0&0&0&\frac{137}{2688}\\0&0&0&0&0&\frac{143}{2835}\\0&0&0&0&0&\frac{3715}{72576}\\0&0&0&0&0&\frac{41}{840}\end{bmatrix}\begin{bmatrix}f_{n-\frac{1}{6}}\\f_{n-\frac{1}{3}}\\f_{n-\frac{1}{2}}\\f_{n-\frac{2}{3}}\\f_{n-\frac{5}{6}}\\f_{n}\end{bmatrix}$$

$$+h\begin{bmatrix}\frac{2713}{15120}&-\frac{15487}{120960}&\frac{293}{2835}&-\frac{6737}{120960}&\frac{263}{15120}&-\frac{863}{362880}\\\frac{47}{189}&\frac{11}{7560}&\frac{166}{2835}&-\frac{269}{7560}&\frac{11}{945}&-\frac{27}{22680}\\\frac{27}{112}&\frac{387}{4480}&\frac{17}{105}&-\frac{243}{4480}&\frac{9}{560}&-\frac{29}{13440}\\\frac{232}{945}&\frac{64}{945}&\frac{752}{2835}&\frac{29}{945}&\frac{8}{945}&-\frac{4}{2835}\\\frac{725}{3024}&\frac{2125}{24192}&\frac{125}{567}&\frac{3875}{24192}&\frac{235}{3024}&-\frac{275}{72576}\\\frac{9}{35}&\frac{9}{280}&\frac{34}{105}&\frac{9}{280}&\frac{9}{35}&\frac{41}{840}\end{bmatrix}\begin{bmatrix}f_{n+\frac{1}{6}}\\f_{n+\frac{1}{3}}\\f_{n+\frac{1}{2}}\\f_{n+\frac{2}{3}}\\f_{n+\frac{5}{6}}\\f_{n+1}\end{bmatrix}.$$

3. ANALYSIS OF BASIC PROPERTIES OF THE COMPUTATIONAL METHOD

In this section, we examine the fundamental characteristics of the proposed computational method. The analyses are carried out following the approaches in Refs. [15, 16].

3.1. ORDER AND ERROR CONSTANT

Definition 3.1: Order of a Computational Method

The linear operator associated with the discrete computational method is explicitly defined as:

$$L\{y(t):h\}=\mathbf{A}^{(0)}\mathbf{Y}_m^{(i)}-\sum_{i=0}^{1}h^i e_i y_n^{(i)}-h^2(d_0 f(y_n)+b_0\mathbf{F}(\mathbf{Y}_m)). \tag{11}$$

$$\begin{aligned}L\{y(t):h\}=&c_0y(t)+c_1hy'(t)+c_2h^2y''(t)+\cdots+c_ph^py^{(p)}(t)\\&+c_{p+1}h^{p+1}y^{(p+1)}(t)+c_{p+2}h^{p+2}y^{(p+2)}(t).\end{aligned} \tag{12}$$

The computational method, together with its associated linear difference operators, is said to be of order p if $\bar{c}_0=\bar{c}_1=\bar{c}_2=\cdots=\bar{c}_p=o$ and $\bar{c}_{p+1}\neq 0$. By definition 3.1, the computational method is of uniform order 7, with error constant given by $c_8=\left[6.7679\times10^{-9}\;\;5.0402\times10^{-9}\;\;5.9803\times10^{-9}\;\;5.0402\times10^{-9}\;\;6.7679\times10^{-9}\;\;6.3790\times10^{-9}\right]$.

3.2. CONSISTENCY

Definition 3.2: Consistency

A computational method is said to be consistent if it satisfies the following conditions:

(i) the order $p\geq 1$,

(ii) $\sum_{j=0}^{k}\alpha_j=0$, and

(iii) $\rho'(1)=\sigma(1)$.

By Definition 3.2, the method is consistent since it has an order $p\geq 1$.

3.3. ZERO STABILITY

Definition 3.3: Zero-Stability

A computational method is said to be **zero-stable** if all the roots $z_s, s=1,2,\ldots,n$ of the first characteristic polynomial $\bar{\rho}(z)$, defined by Refs. [2, 17]:

$$\bar{\rho}(z)=\det\left[zA^{(0)}-E\right], \tag{13}$$

satisfies $|z_s|\leq 1$, and every root with $|z_s|=1$ has multiplicity not exceeding the order of the differential equation as $h\to 0$ Moreover, as $h\to 0$, $\rho(z)=z^{r-\mu}(z-1)^{\mu}$, where μ denotes the order of the differential equation and r is the order of the matrices $A^{(0)}$ and E. The primary implication of zero-stability is its role in controlling the propagation of error throughout the integration process.

By definition 3.3, for the computational method, the first characteristic polynomial is expressed as follows:

$$\rho(z)=z\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}-\begin{bmatrix}0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\end{bmatrix}=\begin{vmatrix}z&0&0&0&0&-1\\0&z&0&0&0&-1\\0&0&z&0&0&-1\\0&0&0&z&0&-1\\0&0&0&0&z&-1\\0&0&0&0&0&z-1\end{vmatrix}=z^5(z-1)=0. \tag{14}$$

Thus, solving for z in

$$z^5(z-1)=0. \tag{15}$$

Gives $z_1=z_2=z_3=z_4=z_5=0$ and $z_6=1$. Hence, the computational method is zero-stable.

3.4. CONVERGENCE

According to Dahlquist's theorem Ref. [18], a computational method is said to be convergent if it is both consistent and zero-stable. Hence, the proposed computational method is zero-stable.

3.5. REGION OF ABSOLUTE STABILITY

Definition 3.5: Region of Absolute Stability

Region of absolute stability is a region in the complex z plane, where $z=\lambda h$, is defined as those values of z such that the numerical solutions of $y'=-\lambda y$ satisfy $y_j\to 0$ as $j\to\infty$ for any initial condition Refs. [2, 18].

To determine the regions of absolute stability of the proposed computational method, we adopt a technique that avoids the direct computation of polynomial roots or the solution of simultaneous inequalities. This approach, known as the boundary locus

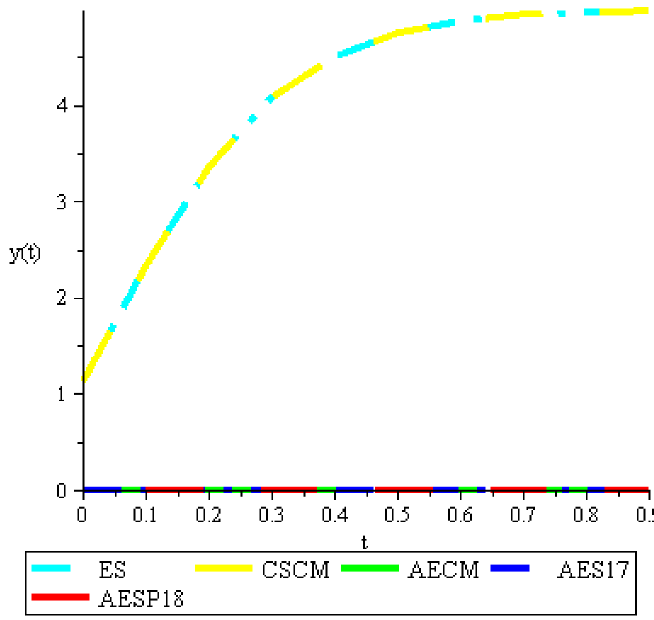

Figure 2. The textual curve of Table 1.

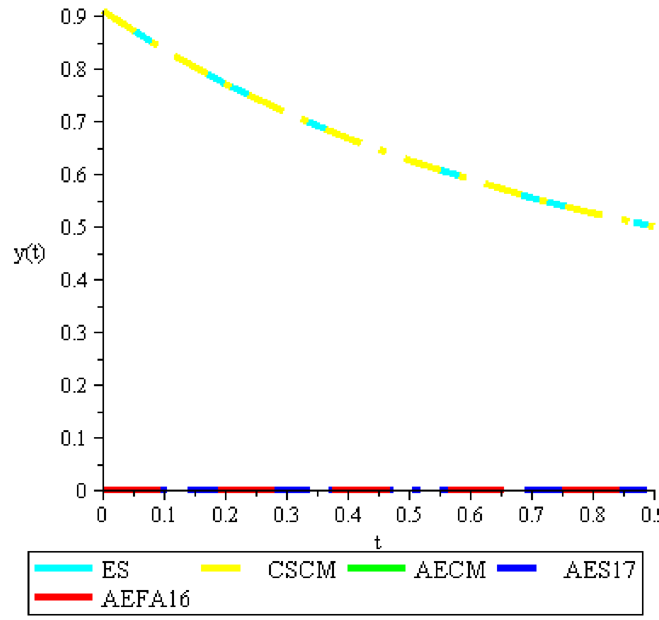

Figure 3. The textual curve of Table 2.

method (BLM), is employed to derive the stability polynomial of the method as:

$$\bar{h}(w) = \left(-\frac{1}{326592}w^5 + \frac{1}{326592}w^6\right)h^6 + \left(-\frac{7}{77760}w^6 - \frac{7}{77760}w^5\right)h^5 + \left(-\frac{29}{19440}w^5 + \frac{29}{19440}w^6\right)h^4 + \left(-\frac{7}{432}w^6 + \frac{7}{432}w^5\right)h^3 + \left(-\frac{25}{216}w^5 + \frac{25}{216}w^6\right)h^2 + \left(-\frac{1}{2}w^6 + \frac{1}{2}w^5\right)h - w^5 + w^6. \tag{16}$$

Using Eq. (16) the region of absolute stability of the proposed computational method is obtained and illustrated in Figure 1:

4. NUMERICAL SIMULATION

We now apply the proposed computational method to some modeled QRDEs of the form given in equation (1.1). The results are computed, and the absolute errors of the proposed method are compared with those reported in Refs. [16, 19, 20]. The acronyms in Table 1 shall be used in Tables and figures.

4.1. PROBLEM 4.1

Consider the QRDE of the form given in Refs. [16, 20]:

$$y'(t) = 10 + 3y(t) - y^2(t), \quad y(0) = 0,$$

whose exact solution is given by:

$$y(t) = -2 + \frac{14e^{7t}}{5 + 2e^{7t}}.$$

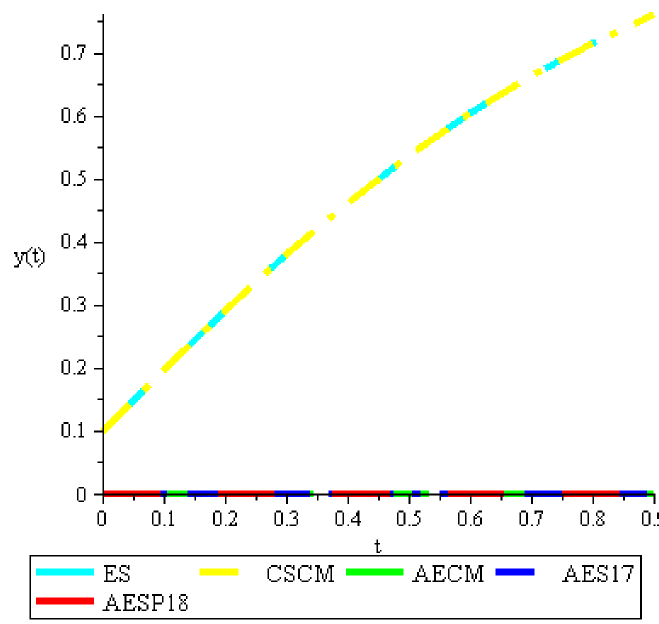

Figure 4. The textual curve of Table 3.

4.2. PROBLEM 4.2

Consider the QRDE of the form Refs. [19, 20]:

$$y'(t) = -\frac{1}{1+t} + y - y^2(t), \quad y(0) = 1,$$

whose exact solution is given by:

$$y(t) = \frac{1}{1+t}.$$

Table 1. Results for problem 4.1.

Notations	Meaning
t	Point of Evaluation for time
ES	Exact Solution
CSCM	Computed Solution in Computational Method (Proposed method)
AECM	Absolute Error in Computational Method
AES17	Absolute Error in Sunday, (2017) Ref. [20]
EFA16	Absolute error in File and Aga, (2016) Ref. [19]
AESP18	Absolute error in Sunday and Philip, (2018) Ref. [16]

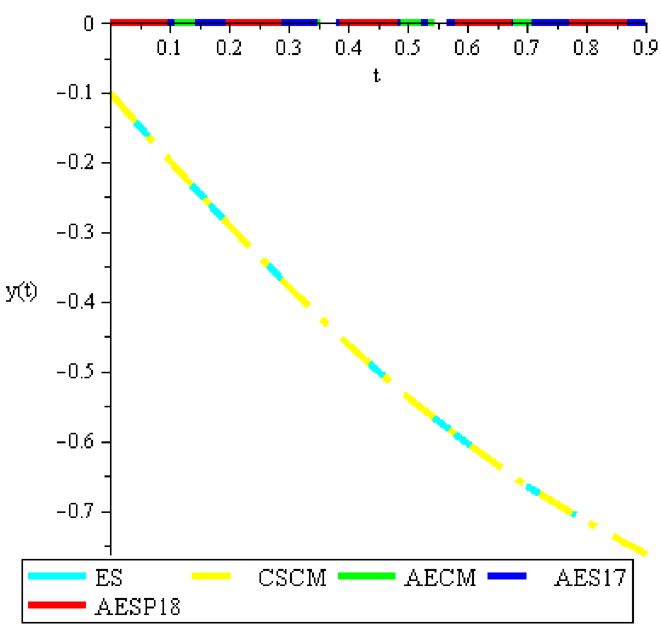

Figure 5. The textual curve of Table 4.

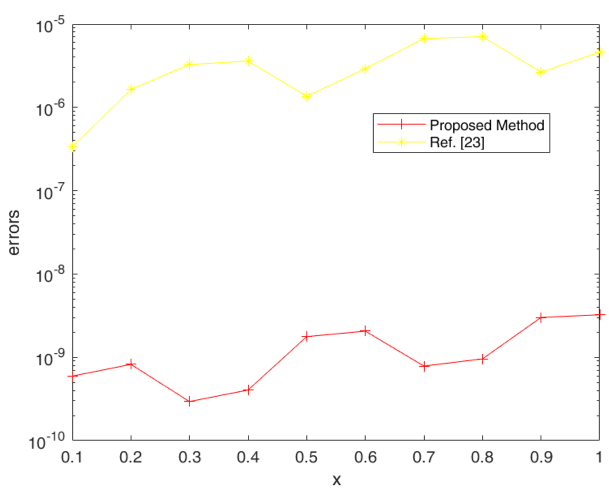

Figure 6. Comparison of the absolute errors for problem 4.1.

4.3. PROBLEM 4.3

Consider the QRDE of the form given in Refs. [16, 20]:

$$y'(t) = 1 - y^2(t), \quad y(0) = 0,$$

with the exact solution:

$$y(t) = \frac{e^{2t} - 1}{e^{2t} + 1}.$$

4.4. PROBLEM 4.4

Consider the QRDE of the form given in Refs. [16, 20]:

$$y'(t) = y^2(t) - 1, \quad y(0) = 0,$$

whose exact solution is given by:

$$y(t) = -\tanh(t).$$

5. DISCUSSION OF RESULTS

Problem 4.1: The QRDE solution obtained using the proposed computational method (denoted as CSCM) is compared against the exact solution (ES). The absolute errors of the computational method (AECM) are consistently smaller than those of other methods across all time points, with especially significant improvements at higher-order time steps. The results reported in Refs. [16, 19] display error trends consistent with earlier methods but with noticeably lower accuracy than the proposed approach. These findings confirm the superiority of the computational method in producing more accurate approximations, particularly as the evaluation point increases. The plots in Figures 2 and 6, based on the data in Table 1, clearly illustrate these differences, with AECM remaining consistently smaller throughout the time interval. Problem 4.2: The results in Table 2 show that the QRDE solution obtained by the computational method (CSCM) closely matches the exact solution (ES), with only minor variations. This high level of accuracy is further confirmed by the AECM values, which remain significantly smaller than those of earlier methods reported in Refs. [19, 20], where errors grow dramatically at larger time values. The gradual increase in AEFA16 highlights the limitations of that method in adapting to varying time steps. In contrast, the error curve of the proposed method remains nearly flat, outperforming the steeper error trends of AES17 and AEFA16, as illustrated in Figure 3. Problem 4.3: Table 3 presents another QRDE test case, where the proposed method again demonstrates superior accuracy. The AECM values are extremely small, even

Table 2. Results for problem 4.1.

t	ES	CSCM	AECM	AES17	AESP18
0.1	1.12295995501998517310	1.12295995587307901600	$8.53094e-10$	$2.82693e-09$	$1.46347e-09$
0.2	2.33036366723934260660	2.33036366744954282610	$2.10200e-10$	$5.89943e-09$	$2.99223e-09$
0.3	3.35929859139218860340	3.35929859155915016660	$1.66962e-10$	$6.83092e-08$	$3.49315e-08$
0.4	4.07625619989394993700	4.07625620020764014650	$3.13690e-10$	$1.49912e-07$	$7.66512e-08$
0.5	4.50864023794231405830	4.50864023797281103320	$3.04970e-11$	$1.83945e-07$	$9.40192e-08$
0.6	4.74705986375186756050	4.74705986386142948480	$1.09562e-10$	$1.65588e-07$	$8.46132e-08$
0.7	4.87206646548954668230	4.87206646560212625910	$1.12577e-10$	$1.24703e-07$	$6.37095e-08$
0.8	4.93588015111826406050	4.93588015119091485170	$7.26508e-11$	$8.43126e-08$	$4.30686e-08$
0.9	4.96801151790818190370	4.96801151794637027740	$3.81884e-11$	$5.32397e-08$	$2.71935e-08$
1.0	4.98407836223863766150	4.98407836225661403400	$1.79764e-11$	$3.21259e-08$	$1.64080e-08$

Table 3. Results for problem 4.2.

t	ES	CSCM	AECM	AES17	AEFA16
0.1	0.90909090909090909091	0.90909090909090234517	6.74574e-15	2.29206e-12	3.82960e-07
0.2	0.83333333333333333333	0.83333333333331878585	1.45475e-14	3.11395e-12	3.82960e-07
0.3	0.76923076923076923077	0.76923076923074948717	1.97436e-14	3.37641e-12	5.79510e-07
0.4	0.71428571428571428571	0.71428571428569152815	2.27576e-14	3.42415e-12	6.81330e-07
0.5	0.66666666666666666667	0.66666666666664226370	2.44030e-14	3.39440e-12	7.33940e-07
0.6	0.62500000000000000000	0.62499999999997472528	2.52747e-14	3.34355e-12	7.60910e-07
0.7	0.58823529411764705882	0.58823529411762131815	2.57407e-14	3.29492e-12	7.74830e-07
0.8	0.55555555555555555556	0.55555555555552953974	2.60158e-14	3.25739e-12	7.82570e-07
0.9	0.52631578947368421053	0.52631578947365798748	2.62231e-14	3.23441e-12	7.87990e-07
1.0	0.50000000000000000000	0.49999999999997356801	2.64320e-14	3.22653e-12	7.93260e-07

Table 4. Results for problem 4.3.

t	ES	CSCM	AECM	AES17	AESP18
0.1	0.09966799462495581711	0.09966799462496078383	4.96672e-15	1.14908e-14	9.71445e-17
0.2	0.19737532022490400073	0.19737532022491326576	9.26503e-15	6.71685e-14	8.32667e-17
0.3	0.29131261245159090582	0.29131261245160304543	1.21396e-14	1.83353e-13	0.00000e00
0.4	0.37994896225522488527	0.37994896225523782456	1.29393e-14	3.38618e-13	2.22045e-16
0.5	0.46211715726000975851	0.46211715726002124437	1.14859e-14	4.86111e-13	2.22045e-16
0.6	0.53704956699803528586	0.53704956699804355047	58.2646e-14	5.79870e-13	1.11022e-16
0.7	0.60436777711716349631	0.60436777711716773622	4.23991e-15	5.94858e-13	3.33067e-16
0.8	0.66403677026784896369	0.66403677026784940031	4.36620e-17	5.32796e-13	5.55112e-16
0.9	0.71629787019902442081	0.71629787019902199420	2.42661e-15	4.16112e-13	5.55112e-16
1.0	0.76159415595576488812	0.76159415595576081059	4.07753e-16	2.74558e-13	3.33067e-16

Table 5. Results for problem 4.4.

t	ES	CSCM	AECM	AES17	AESP18
0.1	-0.0996679946249558171	-0.09966799462495581510	2.02000e-18	1.14769e-14	6.93889e-17
0.2	-0.1973753202249040007	-0.19737532022490399719	5.50000e-18	6.71407e-14	8.32667e-17
0.3	-0.2913126124515909058	-0.29131261245159090165	4.17000e-18	1.83409e-13	0.00000e00
0.4	-0.3799489622552248853	-0.37994896225522488160	3.67000e-18	3.38563e-13	2.22045e-16
0.5	-0.4621171572600097585	-0.46211715726000975622	2.28000e-18	4.86111e-13	1.11022e-16
0.6	-0.5370495669980352859	-0.53704956699803528530	5.60000e-19	5.79869e-13	2.22045e-16
0.7	-0.6043677771171634963	-0.60436777711716349718	8.70000e-19	5.94747e-13	2.22045e-16
0.8	-0.6640367702678489637	-0.66403677026784896536	1.68000e-18	5.32907e-13	5.55112e-16
0.9	-0.7162978701990244208	-0.71629787019902442264	1.83000e-18	4.16001e-13	4.44089e-16
1.0	-0.7615941559557648881	-0.76159415595576488966	1.54000e-18	2.74447e-13	2.22045e-16

smaller than those for Problem 4.2, showing excellent performance for mildly stiff equations. These values are also considerably smaller than AES17 and comparable to the very small errors of AESP18 [16]. This comparison suggests that while both recent methods perform well, the proposed approach has a slight numerical advantage. Visualizations from Table 5 show nearly overlapping error trends between AECM and ESP18, with AES17 trailing due to larger deviations at all evaluation points (see Figure 4). Problem 4.4: The results summarized in Table 4 further confirm the accuracy of the proposed method. Among all problems considered, the AECM values remain the smallest, demonstrating the efficiency of the method in handling QRDEs with decaying solutions. Compared to AES17 and ESP18, the computational method maintains superior accuracy across increasing time points. Figures corresponding to Table 4 show that the AECM error curve is stable and flat, while AES17 and AESP18 display gradual increases, suggesting minor numerical instabilities in these older methods (Figure 5). Overall, these results demonstrate that the proposed computational approach consistently outperforms earlier methodologies in solving various forms of QRDEs, offering a clear advantage in terms of accuracy, stability, and reliability.

6. SUMMARY AND CONCLUSION

This article has presented the development and implementation of a one-step block method for solving quadratic Riccati differential equations (QRDEs). The method was constructed using a combination of collocation and interpolation techniques, with power series approximations employed to derive both the continuous and discrete forms. It was specifically designed to address the challenges posed by nonlinear differential equations, particularly with respect to zero-stability, convergence, and computational efficiency. Numerical simulations demonstrate that the proposed method consistently achieves higher accuracy than existing approaches. The one-step block method has proven effective in preserving both stability and accuracy across a range of test problems, while also reducing computational effort. Its ability to efficiently handle the nonlinear nature of QRDEs highlights its practical value.

The findings of this study confirm the robustness and efficiency of the proposed approach. Its zero-stability, convergence, and ease of implementation position it as a valuable addition to the collection of numerical techniques for differential equations. Future research may focus on extending this method to broader classes of differential equations or refining the computational algorithms for higher-dimensional and more complex systems. Such advancements will enhance its applicability in fields requiring highly accurate solutions to nonlinear problems, including engineering, physics, and biological modeling.

DATA AVAILABILITY

We do not have any research data outside the submitted manuscript.

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